## Elements in the simulation with

## NEGF applied to quantum transport model

1	ASPECT OF THE USER	CASE/SYSTEM TO BE SIMULATED
1.1	ASPECT OF THE USER	Calculation of quantum transport of electrons and holes in semiconductor devices based
	CASE TO BE	on nanostructure. Electrical properties such as current density and charge densities are to
	SIMULATED	be calculated taking into account applied bias and material and geometrical properties of
	AND HOW IT FORMS A	the device. These properties will be used to calculate the flow of electrons through the
	PART OF THE TOTAL	device.
	USER CASE	
1.2	MATERIAL	Nano-scale transistor with InGaAs channel.
1.3	GEOMETRY	<picture be="" document="" final="" in="" included="" to=""></picture>
1.4	TIME LAPSE	Not applicable
	MANUFACTURING	$f^B(E, E_{fB})$ Fermi-Dirac distribution function of the electrons in contact B with the Fermi
1.5	PROCESS OR IN-	level $E_{\mathrm{fB}}$ . The applied bias determines the shift of the contact Fermi levels with respect to
	SERVICE CONDITIONS	their equilibrium position.
1.6	PUBLICATION	

2	GENERIC PHYSICS OF	PHYSICS OF THE MODEL EQUATION			
2.0	MODEL TYPE AND	Non-Equilibrium Green's Function Model for Transport of Electrons (1.4 in RoMM)			
2.0	NAME				
2.1	MODEL ENTITY	The entity in this materials model is electrons expressed by Green's functions.			
2.2	MODEL PHYSICS/ CHEMISTRY EQUATION PE1	Equation	Green's function equations (E-H)G(E)=I  This equation applied for quantum transport is often combined with the MR given below. But here we give the most generic form of the PE.		
		Physical quantities	E Electron energy H Hamiltonian matrix representing the device S Overlap matrix between all involved atomic orbitals		
2.2.1	MATERIALS RELATION 1 MR1 TO PE1				

Kommentar [AdB1]: I am not sure whether this appears in the most generic form of the Green's Function Equation.

# Physical quantities/ descriptors

Hamiltonian in NEGF described using localised basis states in a matrix equation; with on-diagonal elements  $H_{ii}$  giving basis state on-site energy (interaction between one atom and itself), including external potential, off-diagonal elements  $H_{ij}$  (interactions with neighbours) giving energy interaction between basis states and  $S_{ij}$  describing overlap between states;  $H_{ij} = \langle \phi_i | H | \phi_j \rangle_i S_{ij} = \langle \phi_i | \phi_j \rangle_where \underline{\phi}_i$  are localised basis states over which H defined.

The generic PE is often combined with the MR given below and then reads

Retarded Green function:

$$[ES - H - \Sigma^{RB}(E) - \Sigma^{RS}(E)]G^{R}(E) = I$$

Lesser/Greater Green's Functions:

$$G^{\gtrless}(E) = G^{R}(E).\left(\Sigma^{\gtrless B}(E) + \Sigma^{\gtrless S}(E)\right).G^{A}(E)$$

 $\Sigma^{RB}(E)$  Boundary retarded self-energy

 $\Sigma^{RS}(E)$  Scattering retarded self-energy

 $\Sigma^{\geq B}(E)$  Boundary lesser/greater self-energy

 $\Sigma^{\geq S}(E)$  Scattering lesser/greater self-energy

 $G^{R}(E)$  Retarded Green's Function

 $G^A(E)$  Advanced Green's Function

 $G^{<}(E)$  Lesser Green's Function

 $G^{>}(E)$  Greater Green's Function

Here the advanced Green's function is defined as :

$$G^A(E) = (G^R(E))^{\dagger}$$

Note that retarded and advanced solutions correspond to outgoing and incoming waves in contacts.

The Green's functions dependencies are:

$$G^{R}(E) - G^{A}(E) = G^{>}(E) - G^{<}(E)$$

Difference between the retarded and advanced Green's functions is equal to the difference between the greater and lesser Green's functions

#### From current conservation the following relations are derived

# First relation between the boundary self-energies

$$\Sigma^{RB}(E) - \Sigma^{AB}(E) = \Sigma^{>B}(E) - \Sigma^{$$

Difference between the boundary retarded and advanced selfenergies is equal to the difference between the boundary greater and lesser self-energies

#### Advanced boundary self-energy:

$$\Sigma^{AB}(E) = (\Sigma^{RB}(E))^{\dagger}$$

Conjugate of boundary retarded self-energy  $\Sigma^{RB}(E)$  is boundary advanced self-energy  $\Sigma^{RA}(E)$ 

#### Second relation between the boundary self-energies:

$$\Sigma^{< B}(E) = -(\Sigma^{RB}(E) - \Sigma^{AB}(E)) \cdot f_{C}(E)$$

$$\Sigma^{>B}(E) = -(\Sigma^{RB}(E) - \Sigma^{AB}(E)) \cdot (f_{C}(E)-1)$$

 $f_c(E)$  Fermi distribution function of the electrons in the contact C. It requires the knowledge of the corresponding

Kommentar [AdB2]: Ok?

	I	
		Fermi level $E_{Fc}$ .
		Scattering self-energies (description of alloy disorder, roughness, impurity, electron-phonon, and carrier-carrier scattering) $\Sigma^{RS}(E) = \mathcal{L}_R(\text{scattering mechanisms})$
		$\Sigma^{< S}(E) = \mathcal{L}_{<}(scattering\ mechanisms)$
		$\Sigma^{>S}(E) = \mathcal{L}_{>}$ (scattering mechanisms) $\mathcal{L}_{R}$ Function relating any scattering mechanism to the retarded self-energy $\Sigma^{RS}(E)$
		$\mathcal{L}_{<}$ Function relating any scattering mechanism to the lesser self- energy $\Sigma^{\mathcal{L}_{>} Function relating any scattering mechanism to the greaterself-energy \Sigma^{>S}(E)$
		First relation between the scattering self-energies $\Sigma^{RS}(E) - \Sigma^{AS}(E) = \Sigma^{>S}(E) - \Sigma^{ Difference between the scattering retarded and advanced self-energies is equal to the difference between the scattering greater and lesser self-energies$
		Advanced scattering self-energy:
		$\Sigma^{AS}(E) = (\Sigma^{RS}(E))^{\dagger}$ Conjugate of boundary retarded self-energy $\Sigma^{RB}(E)$ is boundary advanced self-energy $\Sigma^{RA}(E)$
		Second relation between the scattering self-energies: $\Sigma^{RS}(E) = -i\frac{\Gamma(E)}{2} + \wp \int \frac{dE'}{2\pi} \frac{\Gamma(E')}{E - E'}$
		$\Gamma(E) = i(\Sigma^{>S}(E) - \Sigma^{ Broadening function\mathscr{D} Cauchy principal integral value Boundary self-energies (coupling of device with its contacts) \Sigma^{RB}(E) = \mathcal{F}_R(H_{DC})$
		$\Sigma^{< B}(E) = \mathcal{F}_{<}(H_{DC}) \bullet f^{B}(E, E_{fB})$
		$\Sigma^{>B}(E) = \mathcal{F}_{>}(H_{DC}) \bullet (f^B(E, E_{fB}) - 1)$
		where: $H_{DC}$ Hamiltonian matrix describing the coupling between the device (D) and its contacts (C) $\mathcal{F}_R$ Function relating $H_{DC}$ and the boundary retarded self-
		energy. Several models exist. $\mathcal{F}_{<}$ Function relating $H_{DC}$ and the boundary lesser selfenergy. Several models exist.
		$\mathcal{F}_{>}$ Function relating $H_{DC}$ and the boundary greater selfenergy. Several models exist.
		And values for the parameters in the above PE <to be="" filled="" in=""></to>
2.4	SIMULATED INPUT	HAMILTONIAN matrix from model1, including details of open contacts
2.6	PUBLICATION ON THIS  ONE SIMULATION	acture of open contacts

3	COMPUTATIONAL I	MODELLING	METADATA		
3.1	NUMERICAL SOLVER	Matrix Alge	Matrix Algebra		
3.2	SOFTWARE TOOL	Within DEEPEN, two in-house tools are used: OMEN (from ETHZ) and TiMeS (from Tyndall) (neither currently supported for external users, but other NEGF codes also available)			
3.3	GRID SIZE	Matrix size determined by number of atoms included in calculation times number of basis states used per atom.			
3.4	TIME STEP	None	None		
	COMPUTATIONAL REPRESENTATION	PHYSICS EQUATION	TISE, Green's function and self-energy equations all expressed in a matrix format.		
3.5	refers to how your	MATERIAL RELATIONS	Green's function, self-energy and Hamiltonian matrix elements calculated by iterative solution of coupled equations.		
	computational solver represents the material, properties, equation variables,	MATERIAL	Hamiltonian, Green's function and self-energy data all stored as floating point values. Both single and double precision are supported but double precision recommended.		
3.6	PUBLICATION	The approach taken in OMEN is overviewed in M. Luisier, "Atomistic simulation of transport phenomena in nanoelectronic devices", Chem. Soc. Rev., 2014,43, 4357-4367. The approach taken in TiMeS is overviewed in D. Sharma, L. Ansari, B. Feldman, M. lakovidis, J. C. Greer and G. Fagas "Transport properties and electrical device characteristics with the TiMeS computational platform: Application in silicon nanowires", J. Appl. Phys. 113, 203708 (2013)			

4	POST PROCESSING	
4.1	THE PROCESSED OUTPUT IS CALCULATED FOR	<ul> <li>Distribution of electron states with respect to energy _with open boundary conditions</li> <li>Spatially- and energetically-resolved density-of-states (DOS)</li> <li>Energy-resolved transmission probability</li> <li>Carrier density</li> <li>Current flow</li> <li>Electrostatic potential all of these are calculated for finite volumes to be used in the continuum model 3.</li> </ul>
4.2	METHODOLOGIES	• Electrostatic potential from the carrier density through the Poisson equation $\nabla \cdot \varepsilon \nabla V = -q(p-n+\sum_i [\delta(\mathbf{r}_{Di}-\mathbf{r})] - \sum_j [\delta(\mathbf{r}_{Aj}-\mathbf{r})])$ with $\varepsilon$ Dielectric constant $\mathbf{r}_{Di}$ Position of $i^{th}$ donor atom $\mathbf{r}_{Aj}$ Position of $j^{th}$ acceptor atom Spatially- and energetically-resolved density-of-states (DOS) calculated through energy integration $\mathbf{r}_{Aj} = -\frac{i}{2\pi} \int dE \ G^{<}(E)$ ; with carrier densities $\mathbf{r}_{Aj} = -\frac{i}{2\pi} \int dE \ G^{<}(E)$ ; with carrier densities $\mathbf{r}_{Aj} = -\frac{i}{2\pi} \int dE \ G^{<}(E)$ ; with electrostatic potential is determined from the carrier density through the Poisson equation • Energy-resolved transmission probability from Green's or Wave Function • Current flow from the transmission probability, through energy integration
4.3	MARGIN OF ERROR	<ul> <li>Generation of energy grid that captures the DOS features, can lead to significant error in the carrier density.</li> <li>Convergence of Poisson equation, needed: &lt;0.1% variations in the</li> </ul>

	electrostatic potential. If not, possible changes in the current by >109	%.